Delineating Store Trade Areas Through Morphological Analysis

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Abstract: The precise knowledge of trade area limits is of importance for companies that want to accurately fit their marketing strategy to local features. Many methods have been already proposed in the literature but either they are too simple or often are at the same time approximate and expensive in computer times. This paper develops the framework of a new method based on mathematical morphology, a science usually used in image processing but not yet applied on data resulting from management sources. This method applied to the delineation of a trade area breaks up the acquisition of the data, filtering, segmentation and regularization of the area boundaries.

Keywords: Filtering, **Location**, Morphological analysis, **Retailing**, Trade areas

Introduction

Managing trade areas is an old marketing problem and a major concern for retail and service firms, especially with the development of retail and service networks. Several methods have been proposed throughout the last century and a trade area mix has been defined many years ago (Rosenbloom, 1976). But how to manage such an area without any precise definition and delineation?

According to Ghosh and McLafferty (1987), the trade area is "the geographic area from which the store draws most of its customers and within market penetration is highest". Huff (1964) describes it as a statistical and more extended concept, "a geographically delineated region containing potential customers for whom there exists a probability greater than zero of their purchasing a given class of products or services offered for sale by a particular firm or by a particular agglomeration of firms".

Many parameters shape the trade area of an outlet or a service firm. These parameters are intrinsic marketing factors of the store i.e. attractiveness, prices, size of the outlet, diversity of the merchandise as well as environmental factors i.e. existence of competitive outlets in the neighborhood, sociological and economical environment (Ghosh and McLafferty,1987). Moreover, trade areas are usually not static but are variable according to time and influenced by space-time factors as local competition, marketing strategy, seasonality or even fashion.

Even though that constitutes a challenge, defining or keeping an eye on trade areas boundaries and specification is strategic for the survival of existing outlets or for projecting the creation of new retail or service firms. In the first case, a trade area analysis serves mainly to continuously adapt the marketing policy to attract as much customers as possible and to maintain and develop its goodwill counterbalancing competitor drawing. In the second case, evaluating trade areas gives the opportunity to judge a business investment at a specific geographic location as well as making sales estimates and determining a future marketing strategy.

I Traditional Methods for Determining Trade Areas

Three classes of methods - theoretical, empirical, and statistical - are used for determining trade area boundaries. Even though the second class with its real world observations is more accurate and convenient to describe some dynamic variations in trade area frontiers, we will first review the most common theoretical methods which are: central places and retail gravity models through the breaking point technique. Then, empirical methods will be revisited before describing statistical methods.

1.1. Theoretical methods

1.1.1. The Central Places Theory of Christaller and the proximal method

According to this theory (Christaller, 1933) within an ideal physical space represented by a uniform distribution of the consumers being able to move uniformly, the location of the outlets is regular and occupies the tops of hexagons. These tops correspond to the points of maximum accessibility for the potential consumers of the trade area. Christaller treats on a hierarchical basis then the points of sale according to their level of importance and shows that the location of a shopping center of higher level (more important turn-over for more customers with a higher requirement) will be optimal at the center of the hexagon formed by six elementary outlets.

The problem is that a market area is often made of non-isotropic distributions of consumers that distort the trade area pattern (Isard, 1956) even if some attempts were done to extend Christaller's theory with the help of geographical transformations and to change from a non-isotropic to an isotropic environment and conversely (Getis, 1963).

The proximal area method assumes that consumers will choose the closest facility to them in accordance with the nearest-center hypothesis of the central-place theory. Trade areas are drawn in constructing Thiessen or Dirichlet polygons (Dirichlet, 1850; Thiessen and Alter, 1911), which are polygon areas closer to a store than to any other stores.

1.1.2. The gravity models

The law of retail gravitation (Reilly, 1931) has been defined from the Newton's law and can be exposed this way: The intermediate population I located between two urban poles A and B will be attracted by each one of these poles in proportion of their size and in opposite proportion of the square of the distances between zone I and cities A and B:

$$\frac{\text{Va}}{\text{Vb}} = \frac{\text{Pa}}{\text{Pb}} \times \frac{Db^2}{Da^2}$$

where V_a and V_b are the proportions of purchases carried out in cities A and B by the inhabitants of the intermediate zone, Pa and Pb are the populations of cities A and B, D_a and D_b are the distances between intermediate zone and the cities A and B.

To delineate the trade area boundary of two distant shopping zones, the population of zones A and B, V_a and V_b , are replaced by the selling surface of the two zones and D_a and D_b are measured by driving times. The breaking point in trade between zone A and zone B is then given by the following value in the x-axis from zone A :

$$x = \frac{\text{Total driving time from zone A to zone B}}{1 + \sqrt{\dfrac{\text{selling surface of zone B}}{\text{selling surface of zone A}}}}$$

The Reilly's law is a deterministic model. Huff (1964) transformed the Reilly's law by making it probabilistic in a sense that a customer can choose a shopping place according to its attraction power determined by two dimensions: its size and the distance between the home and the shopping place. Nakanishi and Cooper (1974) extended the model to an infinity of variables and made it simpler by transforming it, through geometric means and logarithms, in a regression model. Subjective data, especially as far as distance is concerned, can also be considered in this model (Cliquet, 1995).

1.2. Empirical methods

1.2.1. The Driving Time Method

This method used by many practitioners assumes that customers are willing to patronize an outlet only according to the distance or the driving time they spend to join it. Among different parameters determining consumer habits i.e. population density, purchasing power, media network importance, it has indeed been numerically proved that the driving time required to reach a set of outlets is highly influential in determining consumer shopping center choice (Brunner and Mason, 1968).

In fact, theoretical methods are often inaccurate for determining the trade area of existing stores compared to a good knowledge of consumers. One of the most popular empirical methods based on previous experience for determining trade area borders is the analog method.

1.2.2. The Analog Method

Let us suppose that a store knows the addresses of its customers (it is easy to get it today through loyalty cards). The customer addresses are then plotted on a map and the density of the dots reveals through a visual inspection the size, shape and character of the store's trade area (Applebaum and Green, 1974). The delimitation of trade areas is traditionally appreciated by taking stages of level of customers in linear progression. In the worst case, the trade area is supposed to be at circular border irradiating, starting from the outlet on a ray R. Thus, one considers the trade area in general as geographical surface gathering X% of the customers (X=80% for example). To forecast the level of market penetration or per capita sales of a given store, some other analog stores similar in terms of socioeconomic environment and marketing characteristics are used as references, hence the name of this popular method. This mode of procedure proves not very precise because it presupposes a regular reduction in the rate of penetration according to the distance at the outlet or a certain homogeneity and a good distribution of the customers. However, the socio-economic irregularities of the population in space, the geographical borders, the characteristics of competition and the marketing policy of the store make that very often the trade areas are not completely compact. The "holes" or discontinuities of the customers within this surface are thus neglected in the preceding method. One also does not think of considering the discontinuity, which takes place between a zone with strong density of customers and a zone with low density being able to be rather abrupt.

1.3. The statistical methods

1.3.1. The Regression Method

The regression method seeks to measure a parameter of performance by correlating it with various socio-economic, environmental and marketing variables. It thus supposes also to have for base a certain number of stores or past studies which one will draw the experiment to measure the coefficients of a straight regression line like :

$$Y = b_0 + b_1X_1 + b_2X_2 + ... + b_nX_n$$

where Y is the performance parameter ; $X_1, X_2, ... , X_n$ the variables and

$b_0, b_1, b_2, ..., b_n$ the coefficients of the straight regression line.

The method is particularly used to forecast the global performance Y of a projected outlet which can be evaluated through this formula fed with local data (Olsen and Lord, 1979 ; Ghosh and McLafferty, 1987). But it can also be used to estimate the market share of zones surrounding a new outlet location for delineating trade areas.

1.3.2. *The Clustering Methods*

A cluster is defined as a set of similar objects (Hartigan, 1975) and the clustering is the process by which discrete objects can be assigned to groups which have similar features. Many algorithms can solve clustering problems, like the *k-means method*, which consists in incorporating geographic zones around mobile centers of gravity centers. One must first define the number k of zones. k centers of gravity are then considered randomly and each geographical point belonging to the geographic space is allocated to the nearest of all k centers of gravity, thus shaping k zones. The real center of gravity of each k zones is determined and the process of allocating each point to the new defined center of gravity starts again and again the calculation of new centers of gravity corresponding to the new k zones. This algorithm loops until the k zones and the k centers of gravity are invariable. A calculation of the variance inside each zone can be made to check if the averages are significantly different from one class to another and if the number of classes is right.

The k-means are actually a generalization of the well-known clustering algorithm of the p-median problem (Weber, 1909). The method has been used to specify trade areas limits of malls (Huff and Batsell 1977) with the implementation of spline functions which consist to minimizing the curve radius in each point of the trade area border function f(x) ($\int [f(x)'']^2 \, dx$ must be minimized) to make it more regular. Thus, reality can be different, and one can expect a fragmented trade area instead of an assumed compact trade area surrounding the outlet. Characteristics of population (density, socio-economic characteristics, concerns); the environment (road infrastructure), marketing factors (competition, corporate strategies) vary in space and these facts explain the distribution of the customers in several areas sometimes non-related. The *maximum-cut* is another clustering method. It consists in splitting a given graph into k clusters but a good splitting rule is usually hard to find.

In all cases, it is necessary to predefine a k number of areas. If k is chosen as the number of competitors in the analyzed region, the clustering method will assign each customer to a specific outlet.

All these methods are not very precise and often based on intuition even if some available data coming from experience can be used. That is the reason why we are proposing and describing in this paper a new method based on a morphological approach.

2. Morphological Analysis : A New Perspective

The application of the mathematical morphology theory to location science aims at mitigating these lacks by rationalizing the concept of a trade area.

Mathematical morphology based on concepts of topology, signal processing, probabilities and graph theory comprises a great number of applications which all concern the real world. The fields interested by this technique are various and can be for example materials science, geology, biology, geography, robotics. The common point of the possible fields of application is that the processed data can be variable in a space of observation of two dimensions or more except for speech recognition that operates in a one dimension world.

Mathematical morphology attempts to analyze information as a global entity. For this reason, this science brought much, as its name shows it, with the pattern recognition of fingerprint; voice, writing, structure of materials, geological, cytological or genetic structure, electronic circuit and thus with the image processing coming from various sources i.e. sound recording, photography, electronic or optical microscopy, satellite images, radar or sonar images, radiography, echography, and so on.

However, why not use the methods of morphology on other data that those acquired by vision or direct recording of the real world, in using the principle of universality of mathematics ? Morphology is indeed quite as ready to process data which result from human sensors (e.g. quantitative or qualitative marketing surveys) as well as from electronic or optical sensors or mixed human and electronic data bases.

This new method based on mathematical morphology can be described in a sequence of stages :

- Data coding and mapping
- Pretreatment of the data: Filtering
- Segmentation of the data

- Thinning and regularization of the trade area borders

2.1. Data coding and representation

Let us suppose that a store knows the addresses of its customers $C_1,...,C_k$. A database of addresses can be built thanks to information obtained for example from:

1- the discount or loyalty cards (large stores, chains);

2- the modes of payment like checks or blue charts (stores, banks);

3- the bulletins of a game especially organized for the occasion;

4- a study of vehicle license plates on the store parking lot;

5- a direct investigation through a survey.

Using first the common empirical method, each consumer address is represented by a point on a graph corresponding to the 2D-geographical plan and some groups of dots are then obtained. The density of these groups varies in the plan according to the concentration of the customers. The clusters of points show the trade areas from which the outlet draws the essence of its customers. The human eye succeeds rather well by visualizing such a chart to delimit the borders of these clusters thanks to its powerful functions of spectral analysis.

The analytical delimitation of the dense zones proves more difficult through mathematical analysis but nevertheless necessary if one wants to know not only well one's customers for future promotional operations (mailing by district) but also to ensure oneself of the good location of one's outlets.

If one refers to the preceding case, the analysis can relate to the data of customer addresses previously evoked or even on the data of frequentation of the outlet. With each of the k customers $C_1,...C_k$, listed, one can respectively associate a frequentation of the outlet $f_1,...f_k$ over one period T which is selected in an adequate way one week, one month, one year... according to the type of the outlet considered.

The data of investigation are of discrete type just like the numerical data graphic, which facilitates their visual representation (this mathematical talk could be established without parallelization with an unspecified visual representation but this approach facilitates its understanding).

Each address of a customer Ci corresponds to a lit point is a black pixel of co-ordinates (xi, yi) in a perpendicular base (OX,OY): i varying from 1 to N and j from 1 to m for a geographical area division analyzed out of N x m small zones (xi,yi). The black pixel (presence of at least a customer) or white (absence of customer) corresponds in this case to a geographical block included in a grid network. The grid network (matrix) is not inevitably the most adequate partition of geographical space for it does not preserve the topologic properties of the real world like the property of connexity (contrary to the hexagonal network).

To improve the representativeness of the trade area, one can make correspond to each pixel a linear level of gray (or color) according either to the number of customers in the zone, or of the sum Σ f = f_{ij} of the frequentations of the outlet by the whole of the customers of the zone (x_i,y_i) over one period T. Other variables can of course be taken into account according to concerns of the analysis, as the turnover or the profitability related to each customer over a period of time.

Pixel white → No customer in the small element of the geographical grid considered,
frequentation 0
⋮
Pixel clear gray → A little more frequentation,
ex. frequentation f_{Max} / 3
⋮
Pixel dark gray→ Still a little more frequentation,
ex. frequentation f_{Max} x 2/3
⋮
Pixel black → Customer or group of customers of the zone having the maximum frequentation among the customers of the stores
frequentation f_{Max}

If one considers for example each pixel of a square matrix 512 x 512, formed of one byte coded data, one has then 256 levels of possible values for each point, the matrix having a total space memory of 256 K bytes.

2.2. Pretreatment of the data: Filtering

The pretreatment of the data of investigation is intended to facilitate the analysis of the data without reducing the quality of available information. Stemming from signal processing, the principal method consists of an undulatory filtering (an image or an investigation into a geographical sector being a two-dimensional wave). One will thus not only seek to accentuate crenellation (stressing of the borders between the zones of various characteristics), but also avoid pollution by atypical data (noise effects) due for example to errors of investigation (e.g. bad administration or keyboarding), to some false answers (e.g. distorts address) or quite simply to marginal answers within the zone of homogeneous characteristics.

Different kind of filtering can be used for our data: Filtering on average, filtering of Nagao, sigma filtering.

2.2.1. Filtering on average

This data processing amounts processing the matrix of data while taking as new value of each point, the average of the point considered agglomerated with its neighbors. In a matrix, each element (except on the edges) accounts 8 adjacent neighbors (Figure 1) just as on an image with square matrix.

Insert figure 1

Thus, the new value f 'i,j of fi,j is:

$$f'_{i,j} = [f_{i,j} + f_{i+1,j+1} + f_{i+1,j} + f_{i,j+1} + f_{i-1,j-1} + f_{i-1,j} + f_{i,j-1} + f_{i+1,j-1} + f_{i-1,j+1}] / 9$$

This process has the disadvantage of not only smoothing the data but smooth also the transitions. It is to be used only when the density of the customers in the trade area is low.

2.2.2. Filtering of Nagao

One of the most used filtering methods is that of Nagao (Orstom, 1998). One considers this time the vicinity of a step of two elements (or more generally of e elements) around the point (i, j) considered, that is to say a submatrix of 5 x 5 = 25 elements (Figure 2).

Insert figure 2

One calculates then for 9 compact configurations of elements of 3 different types (Figure 3), the average and the variance of the element values.

Insert figure 3

The new value chosen for the point (i, j) is then that of the average corresponding to the smallest variance. This treatment is carried out by considering in turn all the points of the frequentation data matrix.

Contours are thus well preserved because smoothing is done only in its tangential direction i.e. in the direction where the modification is the least visible. This is why it is said that the filter of Nagao is described as a smoothing filter which conserves and even stresses contours. Filtering can be reiterated several times on the already treated matrix. It is observed that this filter is practically idempotent. At the end of some iterations, the image (or the matrix) does not almost change. One stops the processing then.

2.2.3. Sigma Filtering

Sigma filtering is especially a filter used for noise reduction. To any point (i, j) of the zone active image of level $f_{i,j}$ is allocated the average of its neighbors whose level belongs to the interval centered in $f_{i,j}$, half-width 2 x Sigma, Sigma being the local variance in the window. However if the number of neighbors belonging to this interval is lower or equal to the number of neighbors V=2L+1 then $f_{i,j}$ will be replaced by the average of its eight immediate neighbors (Figure 1). This treatment is well adapted to the filtering of an impulse noise by choosing sigma $\sigma<2$ and L=1 (V=2L+1= 3, corresponding to a vicinity 3x3).

Before continuing the description of the processing on the matrix, we will evoke certain useful concepts on the basic morphological transformations.

2.3. Segmentation of the data

2.3.1. Morphological transformations basis

Let X be a related set of binary data forming a matrix (value 0 or 1 corresponding to switched off or switched on points in a space of 2 dimensions), a first possible transformation of this set is the binary dilatation (Figure 4), which causes to increase the total surface of this set. It tends to connect the disjoined parts and to smooth contours.

Insert figure 4

Another morphological transformation is the binary erosion (Figure 5) which smoothes also surface but on the contrary tends to decrease the total surface of the set.

Insert figure 5

To obtain more regular contours, it is possible to carry out a sequence of erosion-dilatation i.e. on the basis of the initial image. One eliminates all the switched on points from the form considered in contact in bottom, top, on the right or on the left with at least a point not belonging to the shape (erosion). Only, therefore remains the points of its interior part. Then, one surrounds each point border of the eroded shape with new points, on the right, on the left, in top, bottom (dilatation).

If one considers instead of binary data, data like the frequentations $f_{i,\ j}$ defined beforehand, these data $f_{i,\ j}$ form a relief which varies in any point defined by its co-ordinates (i, j) according to the value $f_{i,\ j}$, (a set of values represented by the matrix $[\mathbf{f}_{i,\ j}]$).

Binary dilatation or erosion can be generalized to parameters evolving over a wide range of data like the frequentations $f_{i,\ j}$ (Coster and Chermant, 1989) : In the erosion transformation, the value of each point is replaced by the lowest value surrounding it except if it has the highest value among all its neighbors. The dilatation is defined the same way, following the principle that a dilatation of a shape is the erosion of its complementary.

2.3.2. Consequences of morphological transformations

Applied to such data in the case of erosion, the areas with minimal values tend to widen their surface and the areas with maximum values decrease. The morphological transformation of closing combines dilatation and erosion (fills the valleys corresponding to the lowest values without transforming the peaks).

One can also quote the morphological transformation of opening, which is the sequence, in this order, of an erosion and a dilatation, of the same size. Lastly, the morphological transformation of the " top hat " (Meyer, 1978; Serra, 1982) is the subtraction of the data of the initial matrix $[\mathbf{f}_{i,\ j}]$ with the data of the closed matrix $[\mathbf{f}^{\mathbf{T}}_{i,\ j}]$. It constitutes a morphological filter that thus underlines contours and is the transformation that we are going to use.

The result matrix is made of binary data corresponding to a value of 1 if the matrix element belongs to the trade area border and 0 conversely.

2.4. Thinning and Regularization of the trade area borders

Clusters of binary elements belonging to the trade area border are thinned. This means that for a cluster of elements characterizing the border, only the medium element enough to specify this border is kept in the matrix whereas the other elements are set to 0.

The points which were computed at the last step are linked according to bézier regular curves: Let consider an ordered continuation of n+1 points of the plan: $\{P_i = (x_i, y_i)\}$ / i = 0 to n, which defines a polygon in n sides called polygon of control. One calls approximation of Bernstein-Bézier of this set of points, the parameterized curve (Lane and Riesenfelf., 1980):

$$p(t) = \sum_{i=0}^{n} \binom{i}{n} t^i (1 - t)^{n-i} p_i$$

We took n = 3 and obtain what is called cubic curves: The cubic curves are defined using four points, two points located on the curve, at its two ends, the two others out of the two tangents, to some extent exerting an attraction on the curve (Figure 6).

Insert figure 6

Let t be a number between 0 and 1, and p(t) an unspecified point of the curve. While varying t from 0 to 1, one obtains the set of the points which constitute the curve. Here are the equations which make it possible to define the curve:

The polynomial $p(t) = a_p t^3 + b_p t^2 + c_p t + p_0$

where $t \in [0, 1]$

and its coefficients:

$$p_1 = p_0 + c_p / 3$$
$$p_2 = p_1 + (c_p + b_p) / 3$$
$$p_3 = p_0 + c_p + b_p + a_p$$

3. An Application to a Mall Trade Area

The frequentations over one year of each customer of a mall in France have been gathered through a database of discount cards. The geographical urban zone around the mall has been divided in a partition of small square areas, each area measuring about 250 square meters. The frequentation of each customer was then allocated to a single square which is the closest to its address. If more than a customer were registered on a specific square, the frequentations of these customers were all added and allocated to the given square.

The frequentation values corresponding to each square were coded: The maximum registered value had a value of 0 which was also the minimum value for a byte (00 in hexadecimal code) and has the value of black in a bitmap image. The square in which no customer were living had a corresponding value of 255 (FF in hexadecimal code) and has the value of white. The intermediate values of frequentation were linearly distributed over a scale between 0 and 255 describing all gray tones from black to white. This process gave us a geographical mapping of the frequentations (Figure 7) as well as a matrix of frequentations which has been filtered thanks to two Nagao filters (Figure 8). A 'top hat' morphological transformation then followed this filtering and this means as described before, a dilatation (Figure 9), an erosion (Figure 10) and a subtraction from the original image (or matrix) (Figure 11).

After a thinning, the closest isolated points were joined, the unclosed curves were cut and a bézier function was used to smooth the contours (Figure 12). A software has been written in Visual Basic to conduct all these previous processing steps.

Insert figures 7, 8, 9, 10, 11, 12

4. Result Analysis

The matrix (equivalent to an image) used for this process had a resolution of 130 x 130 pixels corresponding to a geographic area analyzed of about 6500 meters x 6500 meters (the space memory of this matrix was 18 kbytes). The data base was composed of more than 60 000 customers.

About 18 fragmented parts are composing the mall trade area. These parts can be ranged in terms of surface: (Figure 12) :

- **Class 1-** 1 wide area corresponding to the highest populated area : area number 1,
- **Class 2-** 1 of a populated area (20 % of area number 1) : area number 2,
- **Class 3-** 7 areas of medium importance (from 4% to 8 % of area 1): areas numbers 3 to 9,
- **Class 4-** 9 areas of small importance (a few % of area 1) : areas numbers 10 to 18.

If the Nagao filter had not been used (Figure 13), less trade area parts would have been detected as seen on the picture. Some closed parts would not also been agglomerated. For instance, the most important part listed before would have been divided up in about 5 or 6 parts.

Insert figure 13

The relative position of each part is an interesting parameter too: One notices that 4 important parts are disposed in a closed neighborhood (Class 1 + Class 2 + 2 areas of class 3) which gathers precisely 80% of the frequentations. It means that if one draws a circle with a center placed at the center of gravity of class 1 and a radius of a few miles, one would certainly include the 80% described before, making Applebaum and Green way (1974) of delineating a trade area a good approximation. The only problem is that this least procedure includes wide geographical parts in which no customers are located. (Figure 14).

Insert figure 14

The morphological approach has a street precision advantage over the other methods. The calculated boundaries defined by geographical coordinates can easily be converted to addresses using a geocoding software. It underlines high customer densities regions whereas the k-means or similar clustering methods only assign customers to k centroïds, creating sets of clients. The following table shows a comparison between possible procedures.

	Type of analysis	Constraints	Advantages	Limitations
Morphological analysis	Based on real data	Needs a database of potential or existing customers locations	Demonstrative and precise (up to the street number description). Shows high density customers areas	Does not take competitors into account at the present development stage
Analog method	Based on real or statistical data	An hypothetical % of customers belonging to the trade area must be predefined	Is a fast approximation if few data are available.	The trade area shape is oversimplied (often a circle)
Driving time method	Theoritical	Driving time estimes in all directions from the outlet	Can be used prior to the store establisment	Lack of precision. Only valid for customers using the car transport mode
Proximal method	Theoritical and geometrical method	Needs a database of all commercial competitors locations	Can be used prior to the store establisment	Lack of precision. Very theoritical with the assumption of an isotropic customers distribution
k-means	Based on real data	The number k of areas is to be specified. Needs a database of customers	Assigns each customer to k different outlets	Does not define a region but only sets of geographical closed customers

These concentrated areas in terms of customers represent the most strategic geographical part for the considered business beside the analog method circle and the driving time trade area which usually gather both concentrated and empty spaces.

We must point out that the trade area analysis made in this paper should take account of natural borders. Although the morphological process usually respects these frontiers, special care has been taken to check out the validity of the data. In particular, a map gathering all natural limits i.e. rivers, forests, parks locations should always be compared with the trade area delineation to check realism of the results and correct them if necessary.

5. Some Managerial and Strategic Benefits

The method described in this paper comprises a certain number of benefits for the management of an outlet. Firstly, the perfect knowledge of the geographical limits of the trade area including the great mass of the customers enable to better target commercial deals and corporate marketing strategy. This geographical customers division makes it also possible to carry out a deeper analysis consisting in correlating the performance in each part of the trade area with various socio-economic, environmental or marketing criteria while being freed from parameters related to the density of population.

It would be desirable indeed in the spirit of this morphological method to compute other mappings where not frequentations but parameters independent of the population like the rate of penetration would be represented and to use the same method to encircle the borders of a good or a bad performance.

In fact, the same process could be applied to various situations as in the case of a retail location search. The geographical limits of potential consumers corresponding to the corporate marketing segments are to be encircled the same way as shown before.

6. Perspective for Future Research

Future research in the field of trade area specification analysis can be led in many directions. We have used morphological transformation to delineate a trade area but other various methods could have been implemented for the same purpose. For example, it is also possible to use topological methods, which are based on the fact that all the data belonging to the same zone of analogous nature are surrounded by the same loop of the gradient. We plan to compare the quality and precision of all available methods with many applications in the real world.

Another direction of future research is to specify exactly the shape of trade areas with morphological properties i.e. perimeter, thickness, compactness, invariant moments and to correlate them to the outlet performance. Discovering some morphological parameters that have an influence on outlet performance in specific cases or in a general manner, would lead the choice of an optimal site for a new retail or a service firm.

A last interesting study would be to observe the variations according to time of trade area shape and features. The analysis should be done in considering the frequentations not over a global period of one year but for example over successive periods of one or two months and then describing the variations of trade areas borders.

Conclusion

The method by mathematical morphology used in this article offers a way of promising research to follow not only the characteristics of a trade area but also for better determining the decision purchase criteria of the consumers. These possibilities formerly reserved to pure sciences like materials mechanics or medicine increase day after day thanks to the development of the data processing which allows the exploitation of expensive mathematical theories in terms of computer time.

Nevertheless, mathematical morphology is undoubtedly more sparing in a number of computer operations per second compared with the algorithms of location-allocation models (Ghosh and Rushton, 1987) since it approaches the marketing analysis according to a total mode while arriving to an immediate solution. It should be mentioned that all the location models developed in the past can be integrated into the morphological approach when focusing them on the inner geographical space determined by this theory.

Indeed, the fact of better describing a trade area makes it possible to concentrate its marketing thought on the aforementioned zone that is to adapt its marketing strategy as well as to consider a new location.

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FIGURE 1

The 8 adjacent neighbors of a central point

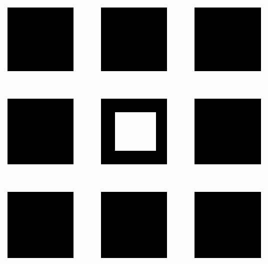


FIGURE 2

The 24 adjacent neighbors of a central point

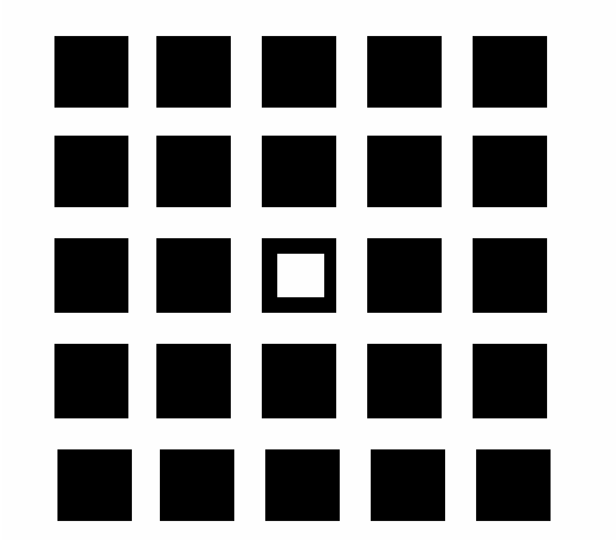
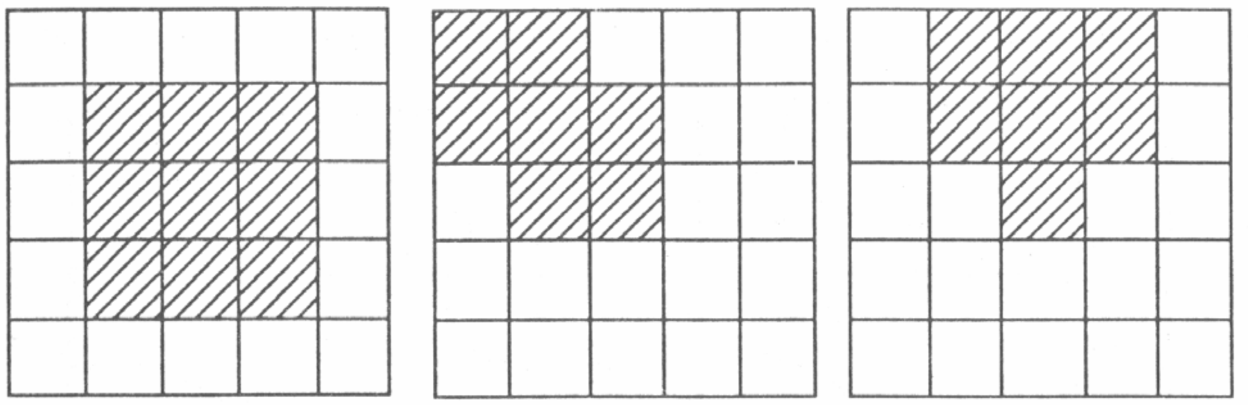


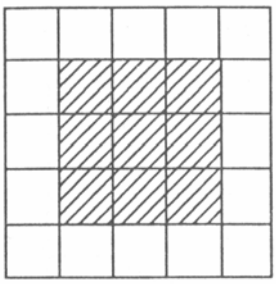
FIGURE 3

The 3 categories of the 9 configurations on which is processed the calculation of the averages and the variances for the filtering of Nagao:

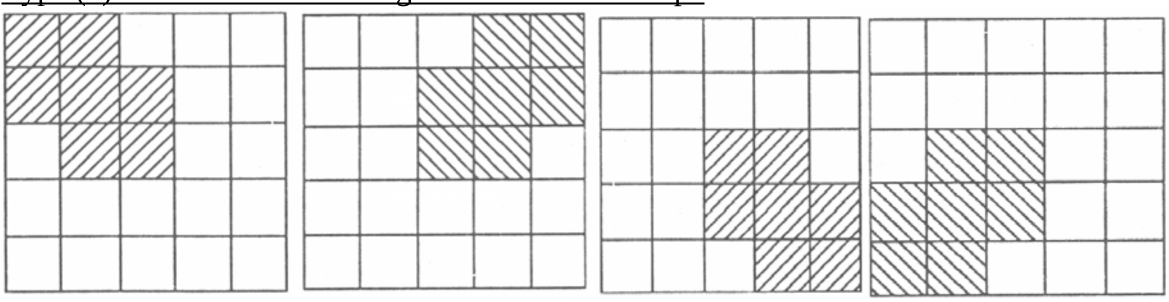
1 of type (1) + 4 of type (2) + 4 of type (3)



Type (1) : 9 values et 1 single configuration



Type (2): 7 values and 4 configurations of same shape



Type (3): 7 values and 4 configurations of same shape

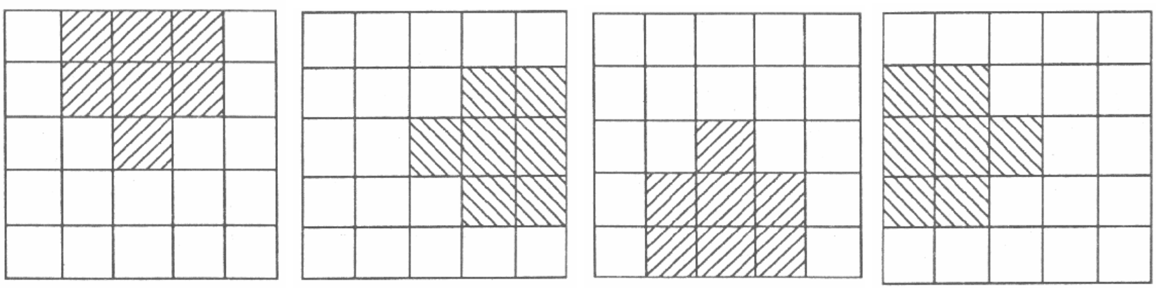


FIGURE 4

Example of a Dilatation : In Hatchings, the Original Shape

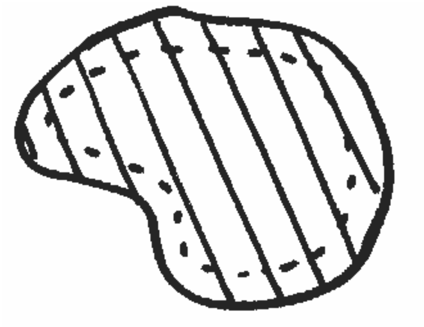


FIGURE 5

Example of an Erosion : In Hatchings, the Original Shape

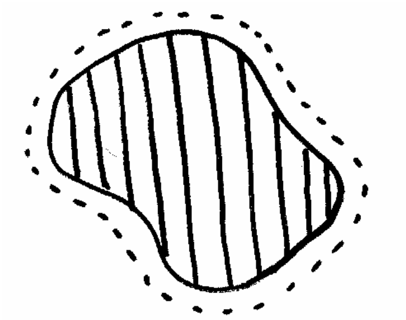


FIGURE 6

The 2 points and the 2 tangents defining a bézier cubic curve

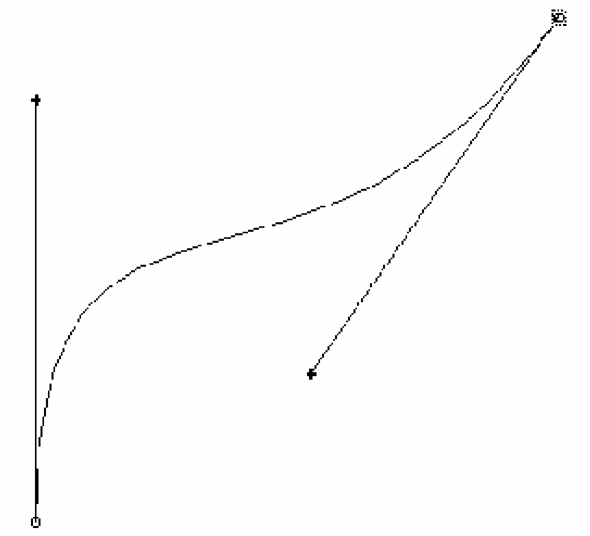


FIGURE 7

The Original Mapping

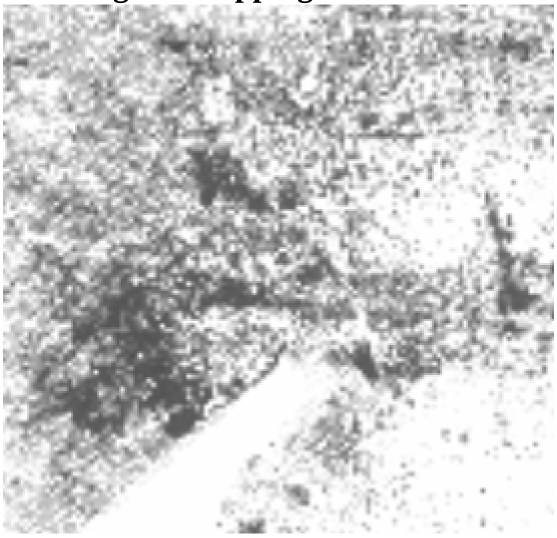


FIGURE 8

A preprocessing with two Nagao Filters

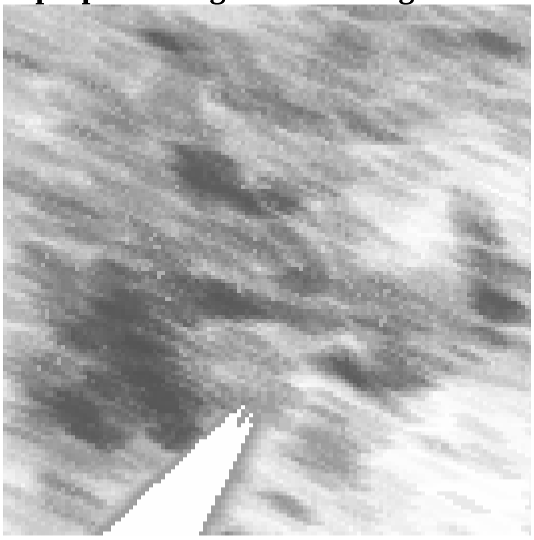


FIGURE 9

Then a Dilatation

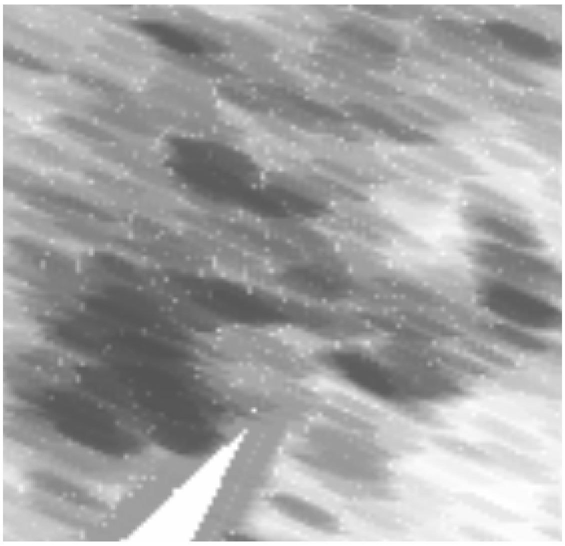


FIGURE 10

Followed by an Erosion

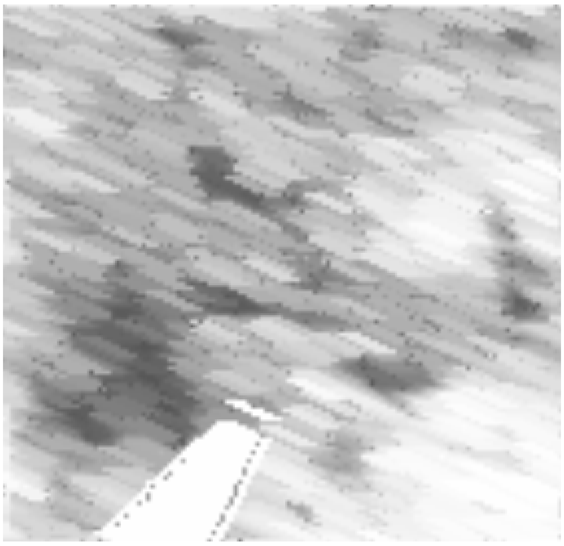


FIGURE 11

A Substraction from the Original Image

FIGURE 12

Thinning and Regularization

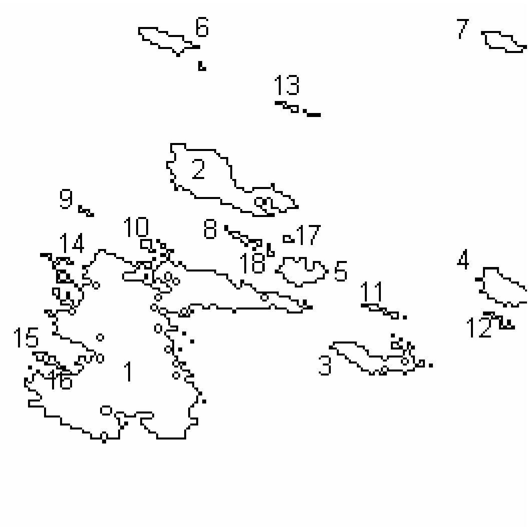


FIGURE 13

The Same Process without a Nagao

FIGURE 14

The trade area boundaries obtained with the morphologic, analog and driving time methods stacked with the customers mapping